

# Asset Pricing in the Resource-Constrained Brain

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## Abstract

Despite scarcity being central to economics, the scarcity of brain's internal resources has largely been ignored. Neuroscience research increasingly points to the brain evolving as a prediction engine in response to this internal-resource scarcity. The brain meets every situation with subconscious expectations, which are contrasted with information to generate error-signals. Selective processing of such error-signals, in lieu of the entire information-stream, saves brain-resources. We show that applying this predictive-processing framework to asset pricing gives rise to an alpha in CAPM. Several empirically observed phenomena correspond to either cross-sectional or time-specific variations in this alpha, potentially synthesizing neoclassical and behavioral finance.

**JEL Classification:** G12, G41

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# Asset Pricing in the Resource-Constrained Brain

Even though resource scarcity has long been a defining notion in economics, the fact that the brain resources (neurons and energy) are also finite has largely been ignored.<sup>1</sup> Perhaps, not having a clear framework for analyzing the implications of such internal resource scarcity has played a role in this neglect. However, over the past decade and a half, neuroscience research has been converging to a framework which views the brain as a ‘prediction machine’ that uses predictions to conserve internal resources.<sup>2</sup> In this article, we show that this framework, known as ‘predictive processing’, provides appropriate conceptual tools for studying the implications of internal-resource scarcity. Specifically, we show that incorporating the predictive-processing framework into asset pricing gives rise to an alpha in the CAPM. Several empirically observed phenomena (value, momentum, size, high-alpha-of-low-beta, and time-specific changes in SML slopes)<sup>3 4</sup> correspond to either cross-sectional or time-specific variations in this alpha. Additional insights about these phenomena emerge that are consistent with empirical evidence. Hence, potentially, a unified explanation for several asset pricing anomalies emerges as ultimately due to the brain’s optimal response to its own internal resource scarcity.

The predictive processing framework says that the brain uses its prior knowledge of the world to form subconscious predictions in every situation. Such predictions are then contrasted with available information to generate error signals. Such error signals are then

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<sup>1</sup> A few exceptions are Alonso et al (2014), Siddiqi and Murphy (2023), and Siddiqi (2023). McKenzie (2018) argues that the neoclassical toolbox extends to behavioral economics if the brain resource scarcity is acknowledged.

<sup>2</sup> There is a large body of literature in the cognitive science that considers the brain to be a prediction machine (Nave et al 2020, Clark 2013, Hohwy 2013, Friston 2010, Bubic et al 2010 among others). A sample based on writings of various cognitive scientists, which is suitable for non-specialist audience, includes Clark (2023), chapter 3 in Hawkins, J. (2021), chapter 4 in Feldman, L. B. (2021a), chapter 4 in Seth, A. (2021), and chapters 4 and 5 in Goldstein (2020). Feldman, L. B. (2021b) also provides a discussion of key ideas.

<sup>3</sup> Fama and French (2016) find deviations from the implications of the model, such as related to beta, size, value, and momentum building on early studies by Black (1972), Stoll and Whaley (1983), Fama and French (1993), and Jegadeesh and Titman (1993) among others. This suggests that there is misspecification in the CAPM, and additional risk factors have been proposed (Fama and French 2016, 2011, 1993).

<sup>4</sup> Specific times when the SML slope is steeper include: Months when inflation is low or negative (Cohen, Polk, and Vuolteenaho 2005), days when news about inflation, unemployment, or Federal Open Markets Committee (FOMC) interest rate decisions are scheduled to be announced (Savor and Wilson 2014), periods of pessimistic investor sentiment (Antoniou et al 2015), and overnight (Hendershott et al 2020).

selectively incorporated into predictions based on the brain's assessment of their relative value. By selectively processing error signals and mostly just leveraging prior knowledge to fill in the gaps, the brain greatly cuts down on the amount of information it needs to process.<sup>5</sup> All expectations, ranging from the mundane (what you expect to see around the corner) to the relatively more sophisticated (risk and reward expectations), are constructed in the brain in this way.<sup>6</sup>

The above description points to the following four components of the predictive-processing framework: (i) an internal model based on a synthesis of prior experiences in similar situations, (ii) subconscious predictions generated by the internal model that reflect typical behavior, (iii) error-signals that result from contrasting predictions with available information, and (iv) importance weights assigned to error-signals and predictions, leading to adjusted predictions, which are consciously experienced.<sup>7</sup>

Applying predictive-processing to asset pricing where a decision-maker (DM) is concerned with equity valuation, requires specifying the above four components. We specify them as follows: (i) the internal model is based on a syntheses of prior experiences with similar firms<sup>8</sup>, (ii) subconscious equity risk and reward expectations (generated by the internal model) that reflect the average behavior in the relevant cluster of firms, (iii) error-signals that are generated by contrasting the available information with the internally generated subconscious expectations, and (iv) importance weights such that the error-signals that generate arbitrage opportunities against the DM are prioritized and eliminated over others, leading to adjusted risk and reward expectations that are consciously experienced.<sup>9</sup>

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<sup>5</sup> See chapter 4 in Hawkins, J. (2021) (and references therein) for a more detailed discussion on the common observation in neuroscience that a lot more brain activity is associated with error-signal processing.

<sup>6</sup> Predictive Processing is more appropriately termed Hierarchical Predictive Processing as it views the mind as being organized in layers with a higher-level layer making predictions about the level just below with only the error-signals reaching the higher-level from the level just below. In this way, the lowest level predicts the incoming sensory signals, whereas a higher level makes predictions about the underlying causes such as changing risk and reward. Hence, this framework offers a unified theory of the mind ranging from sensory perception to higher cognition (see Clark (2013)).

<sup>7</sup> For illustrations of how these components work together to create various experiences, see the appendix in Clark (2023).

<sup>8</sup> This is in line with how internal models are constructed in general. For example, your internal model of a pet dog is constructed based on your prior experiences with pet dogs.

<sup>9</sup> Elimination of arbitrage opportunities implies that a pricing kernel exists which prices all assets as per the fundamental theorem of finance.

While analyzing a firm, the DM's brain activates a relevant internal model (constructed by synthesizing prior experiences with similar firms), which generates subconscious risk and reward expectations that reflect the average behavior in the relevant cluster of firms. These initial expectations are compared with available information about the firm's cashflows to generate error-signals. The resource-constrained brain, in general, does not have sufficient resources to process all errors-signals. So, it prioritizes high value error-signals (errors that create arbitrage opportunities against the DM) over others for processing. This leads to adjusted final expectations that are consciously experienced. In general, expectations are adjusted towards rational expectations to the extent that (exploitable) arbitrage opportunities are eliminated without achieving full convergence. Hence, the influence of internally generated subconscious expectations remains. It is this influence which shows up as the alpha term in the CAPM.

The key novel insight is that the relative resource allocation between processing of risk error-signals vs. reward error-signals matters. We show that if the resources are diverted away from the processing of risk error-signals to reward error-signals than the slope of the security -market-line (SML) flattens. The opposite happens if the diversion is in the reverse direction. The observed times where the SML slope is steeper such as months when there is weak inflation data or deflation indicating higher macroeconomic risks (Cohen et al 2005), on macroeconomic announcement days (Savor and Wilson 2014), periods of pessimistic investor sentiment (Antonioni et al 2015), and around market open (Hendershott et al 2020) when highly leveraged intraday traders typically enter the market, align well with this basic insight.

If the brain pays more attention to the processing of reward error-signals than risk error-signals, then high-alpha-of-low-beta effect or betting-against-beta (BAB) effect arises (Frazzini and Pederson 2014, Black 1972) which gets stronger with the resource tilt favoring reward. When cross-sectional variability in firm profits is higher, it makes sense for the brain to allocate more resources to reward error-signal processing. Similarly, when cross-sectional variability in firm risk is higher, the brain responds by allocating more resources to risk error-signal processing. It follows that BAB and the profitability factor should have a positive correlation, whereas BAB and the investment factor should be negatively correlated. The empirical findings are consistent with this prediction (Novy-Marx and Velikov 2022).

If the resources allocated to the processing of reward error-signal processing are not just larger but sufficiently larger, then the size effect also emerges. Intuitively, subconscious expectations (being an average of all firms in the cluster) overestimate both reward and risk for small-size firms. If reward error-signal processing is much stronger in the DM's brain, then ultimately, risk overestimation dominates in alpha, leading to the size effect. One expects to see this for high quality firms, where available information mostly shows stable profitability without any red flags showing risk related concerns. This prediction matches the empirical findings on the size effect (Asness et al 2018).

Value effect emerges in the brain-based framework as two firms with identical fundamentals may belong to different clusters; hence, have their initial subconscious expectations generated by different internal models. As the value effect is rooted in inter-cluster variation in internal models, it follows that when reliance on internal models is weaker, that is, when the brain assigns low importance weights to predictions coming from the internal models, value effect is weaker. So, one expects to see a weaker or insignificant value effect when major market movements indicate a significant break from the past. This prediction is consistent with the empirical findings on the disappearance of the value effect during the dot.com bubble of the 90's, GFC-2008, and during the Covid-19 pandemic (Campbell, Giglio, and Polk 2023).

Firms that have shown substantial deviation from the norm, such as recent substantially superior or inferior performance, may see a shift in importance weights assigned to error-signals vs internally generated subconscious expectations. This weakening of the importance given to internal models in favor of error-signals generates price momentum. Hence, the brain-based approach predicts that the momentum effect is not only negatively correlated with value, but also is ultimately driven by changing fundamentals, which is consistent with empirical findings on the momentum effect (Novy-Marx 2015).

## 2. The Brain-Based Capital Asset Pricing Model

We rely on a standard derivation of CAPM (for example, as in Frazzini and Pedersen (2014)) and consider an overlapping generations (OLG) economy. The only innovation is that we use the predictive processing framework to specify expectations, which makes rational expectations a special case instead of the only case. Each agent lives for two periods. Agents that are born at  $t$  aim to maximize their utility of wealth at  $t + 1$ . Their utility functions are identical and exhibit mean-variance preferences. They trade securities  $s = 1, \dots, S$  where security  $s$  pays dividends  $d_t^s$  and has  $n_s^*$  shares outstanding and invest the rest of their wealth in a risk-free asset that offers a rate of  $r_F$ .

The market is described by a representative agent who is a mean-variance maximizer:

$$\max n' \{E_t(P_{t+1} + d_{t+1}) - (1 + r_F)P_t\} - \frac{\gamma}{2} n' \Omega_t n$$

where  $P_t$  is the vector of prices,  $\Omega_t$  is the variance-covariance matrix of  $P_{t+1} + d_{t+1}$ , and  $\gamma$  is the risk-aversion parameter.

It follows that the price of a security,  $s$ , is given by:

$$P_t^s = \frac{E(X_{t+1}^s) - \gamma \text{Cov}(X_{t+1}^s, X_{t+1}^M)}{1 + r_F} \quad (2.1)$$

where security  $s$  payoff is  $X_{t+1}^s = P_{t+1}^s + d_{t+1}^s$

and the aggregate market payoff is:

$$X_{t+1}^M = n_1^*(P_{t+1}^1 + d_{t+1}^1) + n_2^*(P_{t+1}^2 + d_{t+1}^2) + \dots + n_S^*(P_{t+1}^S + d_{t+1}^S).$$

As discussed in the introduction, we apply the predictive processing framework as follows: An internal model (trained on prior experiences with similar firms) generates subconscious risk and reward expectations that reflect the average behavior in the cluster. That is, the DM is not aware of the formation of such subconscious expectations. Nevertheless, they play a critical role in the formation of adjusted expectations that are consciously experienced.

Using  $q$  as the cluster identifier and denoting the number of firms in the cluster by  $N_q$ , the subconscious reward and risk expectations are:

$$E^q = \sum_{i=1}^{N_q} \frac{E[X_{t+1}^i]}{N_q} \quad (2.2)$$

$$Cov^q = \sum_{i=1}^{N_q} \frac{Cov[X_{t+1}^i, X_{t+1}^M]}{N_q} \quad (2.3)$$

The brain contrasts these subconscious expectations with available information to generate error-signals. Based on the brain's assessment of their relative importance, error-signals are prioritized for incorporating into expectations. In particular, error-signals that create exploitable arbitrage opportunities against the DM are prioritized over others. In general, in a resource-constrained brain, the initial subconscious expectations are adjusted in the direction of rational expectations without achieving full convergence. This process, which leads to adjusted expectations that are consciously experienced, is described by introducing a parameter,  $m_1$ :

$$E'(X_{t+1}^S) = E^q - m_1 D_1 \quad (2.4)$$

where  $D_1 = E^q - E(X_{t+1}^S)$  is the correct adjustment needed, and  $m_1$  is the fraction of correct adjustment reached so  $0 \leq m_1 \leq 1$ . Rational expectations,  $E'(X_{t+1}^S) = E(X_{t+1}^S)$ , correspond to processing of all error-signals and achievement of full adjustment:  $m_1 = 1$ .

Similarly, the adjusted risk expectation is:

$$Cov'((X_{t+1}^S, X_{t+1}^M)) = Cov^q - m_2 D_2 \quad (2.5)$$

where  $D_2 = Cov^q - Cov((X_{t+1}^S, X_{t+1}^M))$  is the correct adjustment needed, and  $m_2$  is the fraction of correct adjustment,  $0 \leq m_2 \leq 1$ , achieved. Rational expectations,  $Cov'((X_{t+1}^S, X_{t+1}^M)) = Cov((X_{t+1}^S, X_{t+1}^M))$ , correspond to elimination of all gaps and achievement of full adjustment:  $m_2 = 1$ .

If the brain has infinite resources, then of course, it can process all error-signals and always form rational expectations; however, a resource-constrained brain prioritizes error-signals that create exploitable arbitrage opportunities against the DM over others, which in

general adjusts expectations in the direction of rational expectations without necessarily achieving full convergence. A simple re-arrangement of (2.4) and (2.5) leads to:

$$E'(X_{t+1}^S) = E(X_{t+1}^S) + (1 - m_1)(E^q - E(X_{t+1}^S)) \quad (2.6)$$

$$Cov'(X_{t+1}^S, X_{t+1}^M) = Cov(X_{t+1}^S, X_{t+1}^M) + (1 - m_2)(Cov^q - Cov((X_{t+1}^S, X_{t+1}^M))) \quad (2.7)$$

The consciously experienced reward and risk expectations,  $E'(X_{t+1}^S)$  and  $Cov'(X_{t+1}^S, X_{t+1}^M)$  in (2.6) and (2.7), follow from the predictive processing framework as applied to asset pricing. Rational expectations are a special case in the framework corresponding to  $m_1 = 1$  and  $m_2 = 1$ .

The predictive processing framework gives rise to an alpha term in the CAPM as proposition 1 shows.

**Proposition 1 (The Brain-Based CAPM) Predictive processing changes the classical CAPM in only one way: an additional term alpha appears whose value depends on the resource allocation decisions in the brain. The brain-based CAPM takes the following form:**

$$E[R_{t+1}^S] - R_F = \alpha_s + (E[R_{t+1}^M] - R_F) \cdot \beta_s \quad (2.8)$$

where  $E[R_{t+1}^S]$  is the expected (gross) return from stock  $s$ ,  $R_F$  is the (gross) risk-free return,  $E[R_{t+1}^M]$  is the expected (gross) return from the aggregate market portfolio,  $\beta_s$  is the beta of the stock  $s$ , and  $\alpha_s$  takes the following form given below

$$\alpha_s = \left( \frac{\bar{\beta}}{w_s} - \beta_s \frac{(m_1 - m_2)}{(1 - m_2)} \right) \frac{(1 - m_2)\delta_M}{m_1} - \frac{(1 - m_1)}{m_1} \left( \frac{\overline{ER}}{w_s} - R_F \right) \quad (2.9)$$

where  $\bar{\beta} = \sum_{i=1}^{N_q} \frac{\rho_i w_i \beta_i}{N_q}$  is the average market-value weighted beta in the cluster,  $w_i = \frac{n_i^* P_t^i}{P_t^M}$ , ( $P_t^i$  is the share price of firm  $i$ ,  $n_i^*$  is the number of shares of firm  $i$  outstanding, and  $P_t^M$  is the price of the aggregate market portfolio),  $\rho_i = \frac{n_i^*}{n_t^*}$ ,  $\overline{ER} = \sum_{i=1}^{N_q} \frac{\rho_i w_i E[R_{t+1}^i]}{N_q}$  is the average market-value weighted expected return in the cluster,  $w_s = \frac{n_s^* P_t^s}{P_t^M}$  is the market-value weight of firm  $s$ , and  $\delta_M = E[R_{t+1}^M] - R_F$ .



**Proof:**

See Appendix A.

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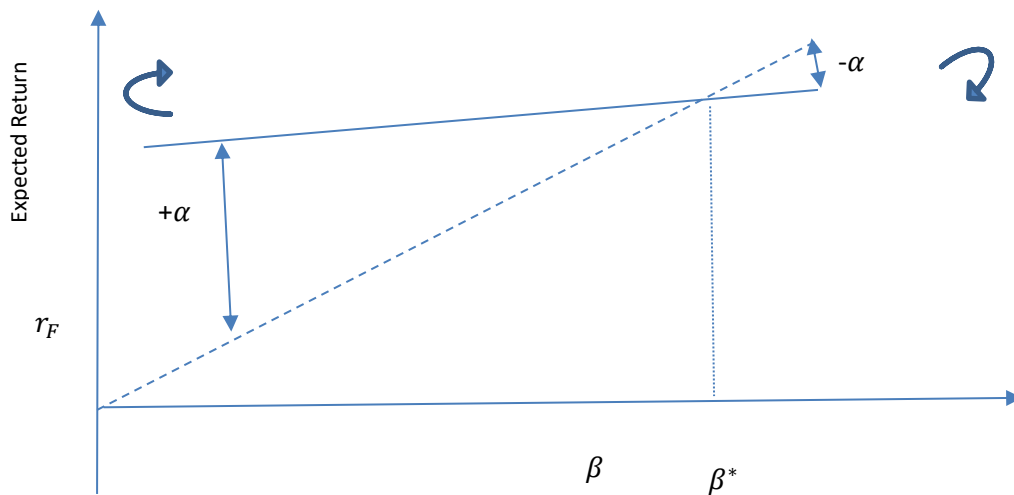
As can be seen from (2.9) in proposition 1, when the DM's brain has sufficient resources to fully process both the reward error-signals and the risk error-signals, that is, when  $m_1 = 1$  and  $m_2 = 1$ , then  $\alpha_s = 0$ , and the classical CAPM is recovered.

### **3. Asset Pricing Anomalies: A Brain-Based Perspective**

The enriched CAPM (presented in proposition 1) has intriguing implications for the slope of the security-market-line (SML). It also generates betting-against-beta (BAB), size, value, and momentum effects, which arise as various partial derivatives of the alpha term in (2.9) depending on resource allocation decisions in the brain between reward error-signal processing vs risk error-signal processing.

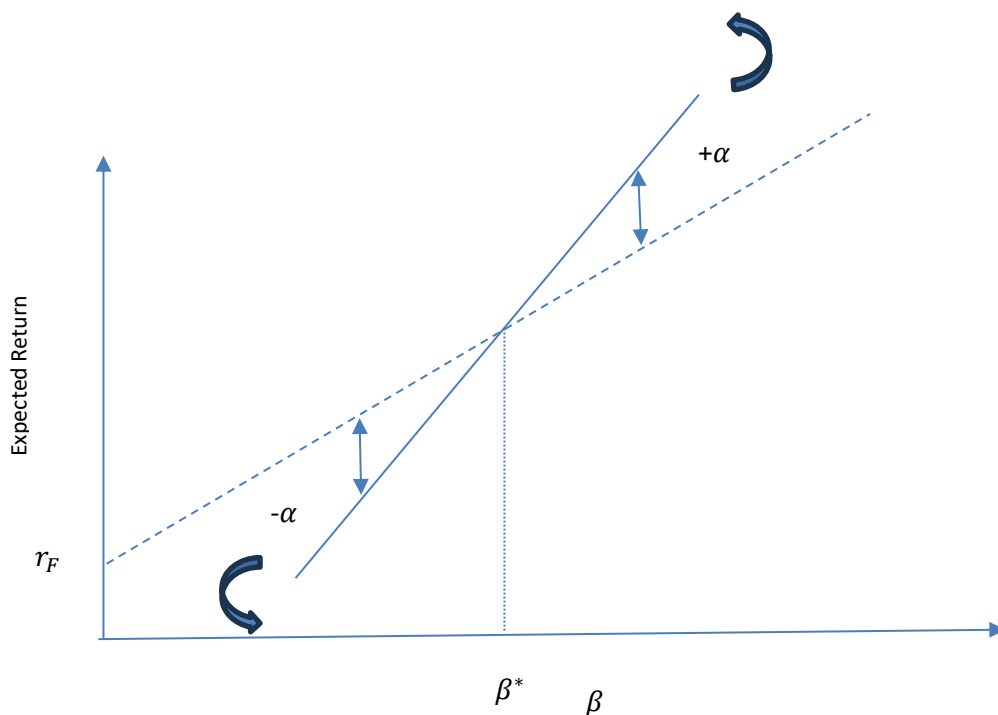
#### **3.1 The Slope of the Security Market Line (SML)**

SML slope depends on the relative resource allocation decisions inside the DM's brain regarding reward error-signal processing vs. risk error-signal processing. If more (less) resources are allocated to reward error-signal processing or less (more) resources are allocated to risk error-signal processing, that is, when  $m_1$  rises (falls) or  $m_2$  falls (rises), then the SML rotates in the clockwise (counter clockwise) direction or the SML slope flattens (steepens). Intuitively, this is due to the changes in the relative underestimation of variation in risk across firms. If it rises, SML flattens, if it falls, SML steepens. Figure 1 and figure 2 illustrate.



**Figure 1 - Slope of the SML when  $m_1$  rises or  $m_2$  falls**

When  $m_1$  rises or  $m_2$  falls, SML rotates in the clockwise direction as there is a threshold value,  $\beta^*$ , below which  $\alpha$  rises (or becomes less negative) and above which  $\alpha$  falls (or becomes more negative). The solid line indicates the brain-based SML whereas the dotted line indicates the classical SML.



**Figure 2 - Slope of the SML when  $m_1$  falls or  $m_2$  rises**

When  $m_1$  falls or  $m_2$  rises, SML rotates in the counter clockwise direction as there is a threshold value,  $\beta^*$ , below which  $\alpha$  falls (or becomes more negative) and above which  $\alpha$  rises (or becomes less negative). The solid line indicates the brain-based SML whereas the dotted line indicates the classical SML.

**Proposition 2 (SML slope)** *If more (less) resources are allocated to reward error-signal processing or less (more) resources are allocated to risk error-signal processing, that is, when  $m_1$  rises (falls) or  $m_2$  falls (rises) then the SML rotates in the clockwise (counter clockwise) direction.*

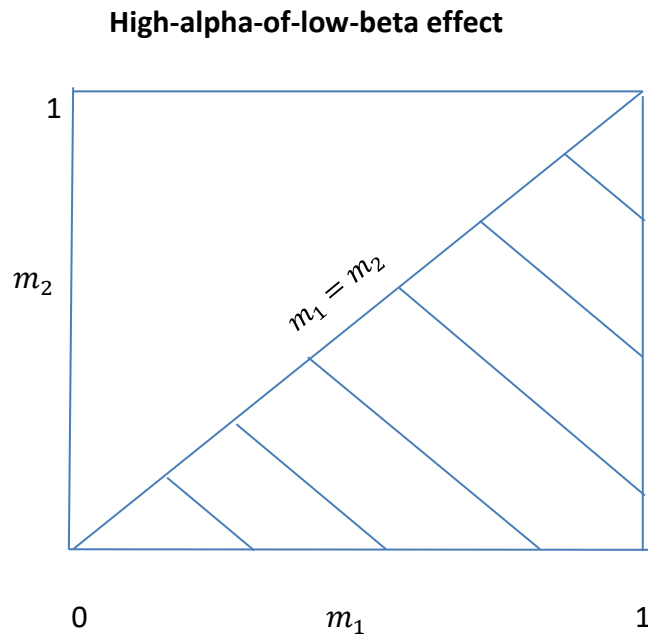
**Proof**

See appendix B.

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The empirically observed variation in SML slope at specific times aligns well with the brain-based model:

- Around market open, the SML slope typically steepens and then gradually flattens during most of the day (Hendershott et al 2020). Intraday traders who are typically highly leveraged enter around market open and then gradually close out their position during the day (Bogousslavsky 2021). Being highly leveraged, such traders' brains assign higher importance weights to risk error-signals. This increases  $m_2$ , which steepens SML as relative underestimation of risk variation across firms falls as a result. SML slope flattens during the day as intraday traders exit the market by closing out their positions for the day, lowering  $m_2$  in the process.
- SML slope is steeper when there is anemic inflation or deflation indicating a weak economy (Cohen et al 2005). It is also steeper in periods of pessimistic investor sentiment (Antoniou et al 2015). It makes sense that the brain gives more importance to risk error-signals during such times. So  $m_2$  rises, which lowers the relative underestimation in risk variation across firms. This steepens the SML slope in the brain-based model.
- SML slope is steeper on macroeconomic announcement days (Savor and Wilson 2014). As most traders have already adjusted their portfolios leading up to the announcement day, trades on the actual announcement day are generally by those whose expectations turned out to be incorrect. The resulting higher importance weights to risk error-signals in the brains of such surprised traders steepens the SML slope ( $m_2$  rises).



**Figure 3** High-alpha-of-low-beta effect is observed in the lined region (with angular lines).

### 3.2 High-alpha-of-low-beta/ Betting-against-beta

In the brain-based CAPM, high-alpha-of-low-beta or betting-against-beta arises under the following condition (taking the partial derivative of alpha in (2.9) with respect to  $\beta_s$ ):

$$\frac{\partial \alpha_s}{\partial \beta_s} = -\frac{\delta_M(m_1 - m_2)}{m_1} < 0 \quad (3.1)$$

Figure 3 shows the region in which high-alpha-of-low-beta or betting-against-beta (BAB) effect is observed in the space of parameters  $m_1$  and  $m_2$ . The effect is observed if  $m_1 > m_2$ .

**Proposition 3 (High-alpha-of-low-beta/Betting-against-beta (BAB))** *High-alpha-of-low-beta effect arises if the importance weights assigned to reward error-signals are higher than the importance weights assigned to risk error-signals such that  $m_1 > m_2$ .*

Overall, the brain-based approach predicts that the high-alpha-of-low-beta effect is not universally observed. The effect is only observed when  $m_1 > m_2$ , and it gets stronger when

$m_1 - m_2$  rises. Specific times when profitability variation across firms is higher may compel the brain to assign higher importance weights to reward error-signals, so  $m_1$  rises, and times when risk variation across firms is smaller may lead to the brain lowering the importance weights on risk error-signals, so  $m_2$  falls. As the profitability factor (Fama and French 2016) is typically constructed by subtracting the average return on weak operating profitability portfolio from the average return on robust operating profitability portfolio, the profitability factor is higher when profitability variation across firms is higher. Similarly, as the investment factor (Fama and French 2016) is constructed by subtracting the average return on aggressive investment portfolio (largest year-on-year rise in assets) from the average return on conservative investment portfolio, lower investment factor generally corresponds to lower risk variation across firms. So, the brain-based model predicts that high-alpha-of-low-beta effect should be positively correlated with the profitability factor and negatively correlated with the investment factor. This is consistent with the empirical findings in Novy-Marx and Velikov (2022). Another novel prediction from (3.1) is that, in periods of falling prices due to higher risk aversion (which leads to higher ex-ante equity premium), the effect is stronger.

### 3.3 The Size Effect

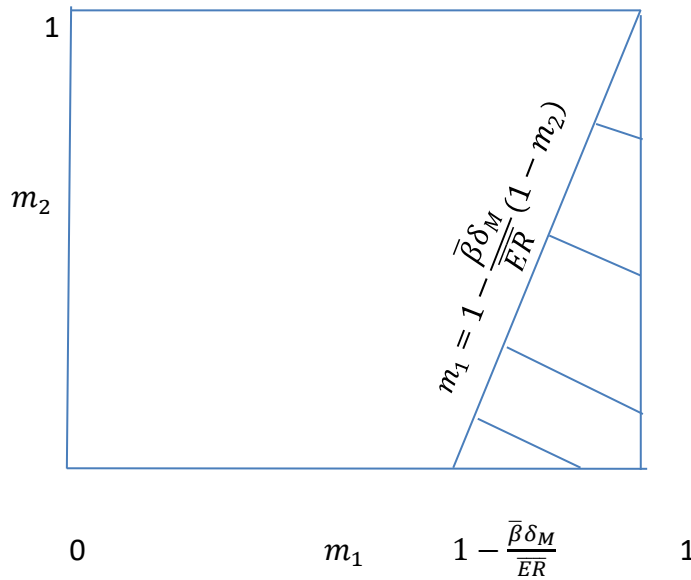
The size effect refers to the notion that small-cap stocks outperform large-cap stocks, all else equal. Taking the partial derivative w.r.t the market-cap weight,  $w_s$ , in (2.9):

$$\frac{\partial \alpha_s}{\partial w_s} = -\delta_M \frac{\bar{\beta} (1 - m_2)}{w_s^2 m_1} + \frac{(1 - m_1) \overline{ER}}{m_1 w_s^2} \quad (3.2)$$

$$\Rightarrow \frac{\partial \alpha_s}{\partial w_s} < 0 \text{ if } m_1 > 1 - \frac{\bar{\beta} \delta_M}{\overline{ER}} (1 - m_2).$$

So, the size effect arises due to resource allocation decisions in the brain if the importance assigned to reward error-signals is sufficiently larger than the importance assigned to risk error-signals such that  $m_1$  is sufficiently larger than  $m_2$ . Intuitively, subconscious expectations (that reflect cluster average) overestimate both the reward and the risk for small-size firms. If  $m_1$  is sufficiently larger than  $m_2$ , then risk overestimation dominates alpha, resulting in the size effect. Figure 4 illustrates.

### The Size Effect

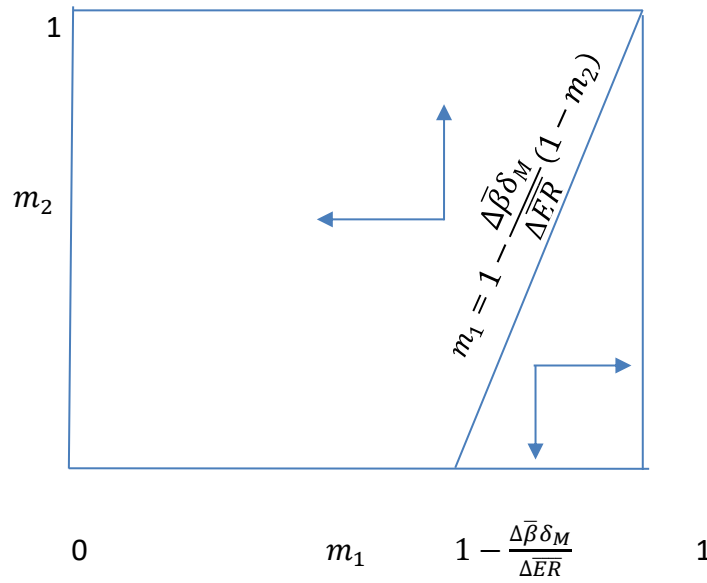


**Figure 4** The size effect is observed in the lined region (with angular lines).

**Proposition 4 (The Size Effect)** *The size effect arises when the importance weights assigned to reward error-signals are sufficiently larger than the importance weights assigned to risk error-signals such that  $m_1 > 1 - \frac{\bar{\beta}\delta_M}{ER}(1 - m_2)$ .*

For firms that have high and steady profitability and low risk, the importance weights assigned by the DM's brain to reward error-signals are likely much larger than the importance weights assigned to risk error-signals. So, size effect is expected to matter among high quality firms. In line with this prediction, empirical evidence shows that the size effect matters for high quality stocks (high profitability and low risk) (Asness et al 2018).

### The Value Effect



**Figure 5** The value effect gets stronger in the direction of the arrows in the two regions split by the line  $m_1 = 1 - \frac{\Delta\bar{\beta}\delta_M}{\Delta ER}(1 - m_2)$ .

### 3.4 The Value Effect

The value effect arises in the brain-based CAPM due to inter-cluster variation in internal models. That is, two firms with identical fundamentals have different alphas (and price to book ratios) if they belong to different clusters with each cluster having its own internal model. Suppose there are two firms,  $a$  and  $b$  with identical fundamentals. However, they belong to different clusters with each cluster having its own internal model. Their alphas are:

$$\alpha_a = \left( \frac{\bar{\beta}_a}{w_s} - \beta_s \frac{(m_1 - m_2)}{(1 - m_2)} \right) \frac{(1 - m_2)\delta_M}{m_1} - \frac{(1 - m_1)}{m_1} \left( \frac{\overline{ER}_a}{w_s} - R_F \right) \quad (3.3)$$

$$\alpha_b = \left( \frac{\bar{\beta}_b}{w_s} - \beta_s \frac{(m_1 - m_2)}{(1 - m_2)} \right) \frac{(1 - m_2)\delta_M}{m_1} - \frac{(1 - m_1)}{m_1} \left( \frac{\overline{ER}_b}{w_s} - R_F \right) \quad (3.4)$$

The difference in their alphas is:

$$\Delta\alpha = \frac{\Delta\bar{\beta}}{w_s} \frac{(1 - m_2)\delta_M}{m_1} - \frac{(1 - m_1)}{m_1} \frac{\Delta\overline{ER}}{w_s} \quad (3.5)$$

As long as  $\Delta\alpha$  is different from zero, the value effect is observed with the low price to fundamentals stock outperforming the high price to fundamentals stock.  $\Delta\alpha = 0$  if  $m_1 = 1 - \frac{\Delta\bar{\beta}\delta_M}{\Delta ER}(1 - m_2)$ . Away from this line, the value effect gets stronger. Figure 5 illustrates.

**Proposition 5 (The Value Effect)** *If the resource allocation decisions in the brain are such that the inter-cluster variation in risk is not exactly balanced by the inter-cluster variation in reward, then the value effect is observed. Specifically, the effect is observed as long as  $m_1 \neq 1 - \frac{\Delta\bar{\beta}\delta_M}{\Delta ER}(1 - m_2)$ .*

It directly follows from (3.5) that the value effect is stronger among small-cap stocks. That is, its magnitude rises as  $w_s$  falls. This provides a theoretical justification for the small-cap value strategy popular among professional traders. As the value effect in the brain-based CAPM has its roots in inter-cluster variation in internal models, it gets weaker if the brain lowers the importance weights assigned to internally generated predictions coming from the internal models. This is likely if there are major market movements suggesting a substantial break from the norm (making past less of an indicator of the future). This prediction is consistent with the empirical findings on the weakness/disappearance of the value effect in unusual time periods such as during parts of the dot.com bubble of the 90's, GFC-2008, and the Covid-19 pandemic (Campbell, Giglio, and Polk 2023).

### 3.5 The Momentum effect

The empirical findings regarding the price momentum show how stocks with superior (inferior) recent performance continue to outperform (underperform) in the short run. In the brain-based framework, the price of a security  $s$  from (2.1) is:

$$P_t^s = \frac{E'(X_{t+1}^s) - \gamma Cov'(X_{t+1}^s, X_{t+1}^M)}{1 + r_F} \quad (3.6)$$



Where:

$$E'(X_{t+1}^S) = E(X_{t+1}^S) + (1 - m_1)(E^q - E(X_{t+1}^S))$$

$$\Rightarrow E'(X_{t+1}^S) = m_1 E(X_{t+1}^S) + (1 - m_1)E^q \quad (3.7)$$

$$\text{And, } Cov'(X_{t+1}^S, X_{t+1}^M) = m_2 Cov(X_{t+1}^S, X_{t+1}^M) + (1 - m_2)Cov^q \quad (3.8)$$

with the superscript  $q$  denoting the subconscious expectations (that reflect the cluster average based on internal models trained on past experience).

In the brain-based CAPM, price momentum arises due to an increase in the importance weights given to error-signals that follow a large change in the fundamentals of momentum winners and losers. To fix ideas, suppose the reward fundamentals of a firm (the momentum winner) improve, so  $E(X_{t+1}^S)$  and consequently,  $E'(X_{t+1}^S)$  goes up, which increases the stock price immediately. The reward fundamentals of another firm (the momentum loser) deteriorate. So, its price falls. This change in fundamentals, then triggers a change in the importance weights given to reward error-signals. So,  $m_1$  goes up. For momentum winners (drawn from top 10% of firms by recent performance), the internal model underestimates reward,  $E^q < E(X_{t+1}^S)$ , whereas for momentum losers (bottom 10% by recent performance), the internal model overestimates reward,  $E^q > E(X_{t+1}^S)$ . So, this increase in  $m_1$ , which follows a large change in fundamentals, increases the price of the momentum winner further and lowers the price of the momentum loser further.

The brain-based model predicts that the price momentum is ultimately a fundamentals-driven phenomena where an initial large change in fundamentals subsequently triggers an increase in importance weights given to error-signals. This prediction is consistent with the empirical findings on momentum effect being fundamentals driven (Novy-Marx 2015). As the increase in importance weights given to error-signals comes at the expense of the importance weights on internal models, momentum and value (which captures inter-cluster variation in internal models) are negatively correlated.

**Proposition 6 (The Momentum Effect)** *Firms with recent large positive changes in earning fundamentals show a further increase in their market prices, and firms with recent large negative changes in earning fundamentals show a further decline in their market prices due to an increase in brain resources allocated to their valuations.*

## 4. Conclusions

Scarcity of resources available in the external world has long been a defining notion in economics. However, another critically important scarcity, which is the scarcity of the brain's internal resources (neurons and energy) has largely been neglected. Perhaps, this neglect is due to a lack of a coherent framework through which the implications of such internal resource scarcity could be analyzed. During the last decade and a half, neuroscience research has increasingly been converging to a framework that points to the brain evolving as a prediction engine in response to this internal-resource scarcity. According to this framework, which is generally referred to as predictive processing, the brain meets every situation with subconscious predictions, which are contrasted with information to generate error-signals. Selective processing of such error-signals, in lieu of the entire information-stream, saves brain-resources. In this article, we apply this framework to asset pricing and show that this gives rise to an alpha in the CAPM. This alpha term generates various CAPM anomalies such as size, BAB, value, and momentum, while leading to a flat SML that steepens at specific times and horizons based on internal resource allocation decisions in the brain. This approach potentially offers a synthesis of behavioral and neoclassical finance as behavioral biases emerge due to the brain's optimal response to its own internal resource scarcity.<sup>10</sup>

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<sup>10</sup> In particular, the anchoring bias in Siddiqi (2019) and Siddiqi (2018), small-risk neglect in Siddiqi and Quiggin (2019), and zero-risk bias in Siddiqi (2017) can all be modelled as directly arising from predictive processing.

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## Appendix A

Substituting from (2.6) and (2.7) into (2.1) and solving for expected return of  $s$ ,  $E[R_{t+1}^s]$ :

$$E[R_{t+1}^s] = R_F + \frac{\gamma}{P_t^s} \left[ Cov(X_{t+1}^s, X_{t+1}^M) + (1 - m_2) \left( Cov^q - Cov((X_{t+1}^s, X_{t+1}^M)) \right) \right] - \frac{(1 - m_1)}{P_t^s} [E^q - E(X_{t+1}^s)] \quad (A1)$$

Multiplying (A1) by the market-value weight,  $w_s = \frac{n_s^* P_t^s}{P_t^M}$ , and aggregating across all firms, one can solve for  $\gamma$  as follows:

$$\gamma = \frac{(E[R_{t+1}^M] - R_F)}{Var(R_{t+1}^M) P_t^M} \quad (A2)$$

Substituting (A2) into (A1) and re-arranging leads to (2.8).

## Appendix B

$$\frac{\partial \alpha_s}{\partial m_1} = \frac{1}{m_1^2} \left[ \frac{\overline{ER} - \bar{\beta}(1 - m_2)\delta_M}{w_s} - \beta_s \delta_M m_2 \right] \quad (B1)$$

There is a threshold value of  $\beta_s$  below which  $\frac{\partial \alpha_s}{\partial m_1} > 0$  and above which  $\frac{\partial \alpha_s}{\partial m_1} < 0$ . It follows that when  $m_1$  rises, SML rotates in the clockwise direction. That is, SML flattens.

$$\frac{\partial \alpha_s}{\partial m_2} = \frac{-\bar{\beta}\delta_M}{m_1 w_s} + \frac{\beta_s \delta_M}{m_1} \quad (B2)$$

There is a threshold value of  $\beta_s$  below which  $\frac{\partial \alpha_s}{\partial m_2} < 0$  and above which  $\frac{\partial \alpha_s}{\partial m_2} > 0$ . It follows that when  $m_2$  rises, SML rotates in the counter-clockwise direction. That is, SML steepens.